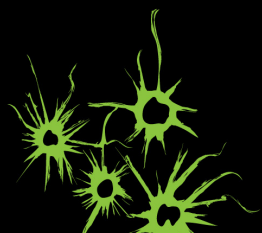
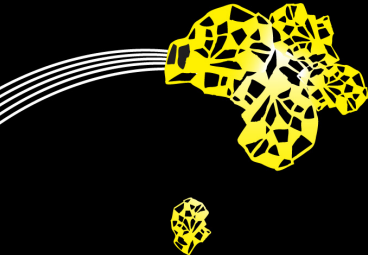


The Potential of Network Coding in Wireless Communications

Jasper Goseling

Stochastic Operations Research Group,
University of Twente

Future Networks Workshop, May 18–19, 2011



Background

Jasper Goseling

- ▶ Ph.D. from Delft University of Technology, Wireless and Mobile Communications Group, 2010
- ▶ Copromotor: Jos Weber, Promotor: Sonia Heemstra de Groot

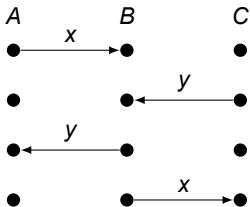
Stochastic Operations Research Group,
University of Twente

- ▶ Department of applied mathematics
- ▶ Stochastic processes and their applications in telecommunications, logistics and health care



Network Coding

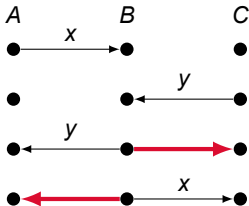
Traditional



- ▶ Wireless network, bits x and y

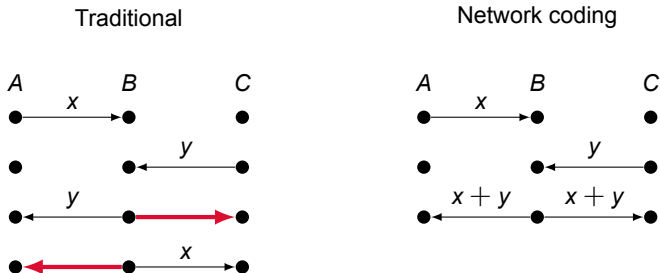
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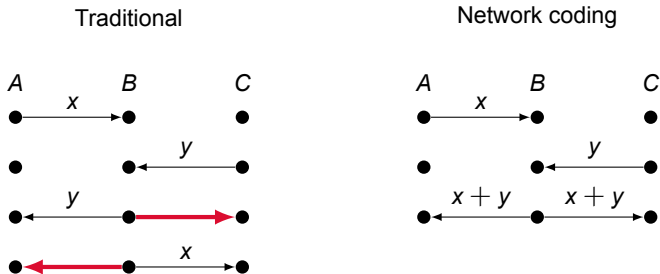
- ▶ Wireless network, bits x and y
- ▶ Transmissions by relay are useful to only one neighbour

Network Coding



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- ▶ Network coding: transmit exclusive or of x and y . C recovers x as $x = (x \text{ XOR } y) \text{ XOR } y$.

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Network coding increases efficiency

“Bits are not Cars” [Ralf Koetter]

00001100



+

+

10101010



=

=

10100110



Interpretations

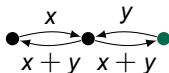
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Interpretations

- ▶ Instead of transmitting the information itself, devices are transmitting “evidence” of this information.
- ▶ Devices collect a system of linear equations in the information bits

$$x + y = 0$$

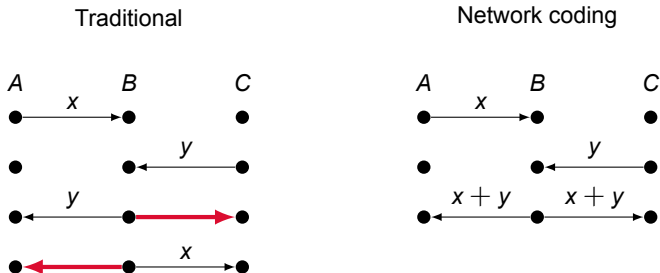
$$y = 1$$



Outline

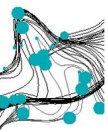
- ▶ Introduction
- ▶ Applications of network coding
- ▶ Energy savings in wireless networks
- ▶ Stochastic arrival and transmit times

Network Coding



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Network coding increases efficiency



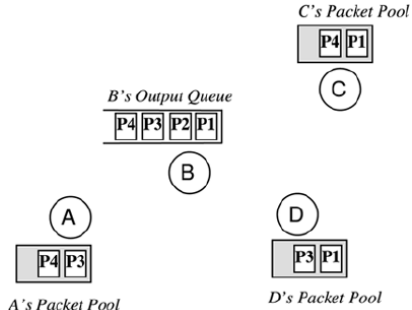
Applications of network coding

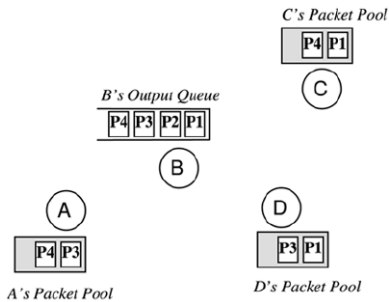
COPE: Wireless Mesh Networks

Katti et al., IEEE/ACM TON,
vol. 16, no. 3, pp. 497–510,
2008

Key aspects:

- ▶ Opportunistic listening
- ▶ Learning neighbour states
- ▶ Opportunistic coding





Packets in B's Queue Next Hop

- P1** → A
- P2** → C
- P3** → C
- P4** → D

Coding Option

Is it good?

- P1** + **P2** Bad Coding (C can decode but A can't)
- P1** + **P3** Better Coding (Both A and C can decode)
- P1** + **P3** + **P4** Best Coding (Nodes A, C, and D can decode)

COPE: Wireless Mesh Networks

Testbed:

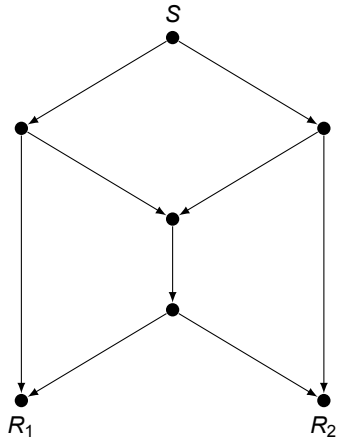
- ▶ 20 nodes, 802.11 ad hoc mode, UDP traffic.
- ▶ Implementation of COPE in between MAC and routing layers.

Result:

- ▶ 3 – 4× increase in throughput.

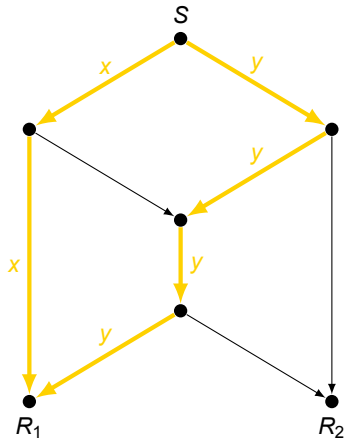


Network Information Flow: Multicast



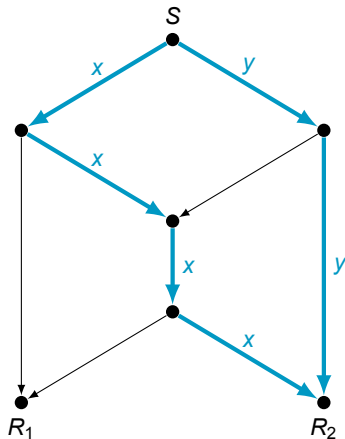
Network Information Flow: Multicast

- Unicast to R_1 or R_2 : Throughput 2.
(Max-flow = Min-cut).



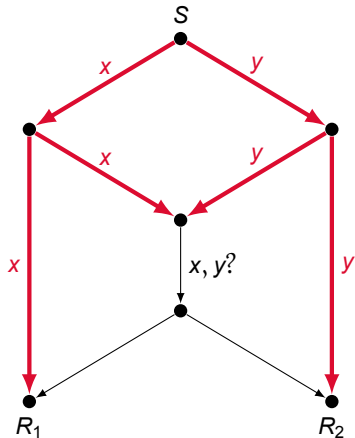
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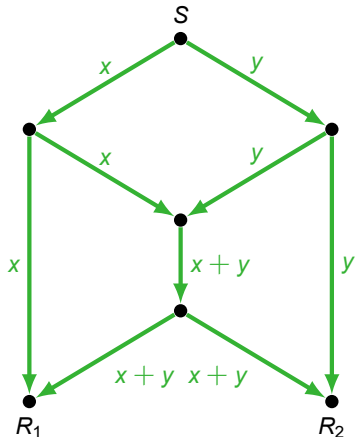
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Network Information Flow: Multicast

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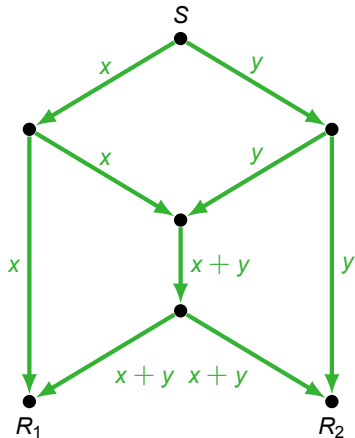


Network Information Flow: Multicast

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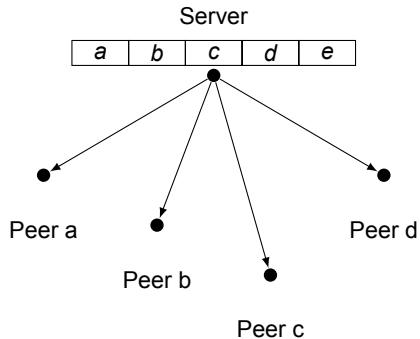
Theorem (Ahlsweede et al. 2000)

Maximum multicast information flow equals minimum of individual min-cuts.



Avalanche: Large-scale Content Distribution

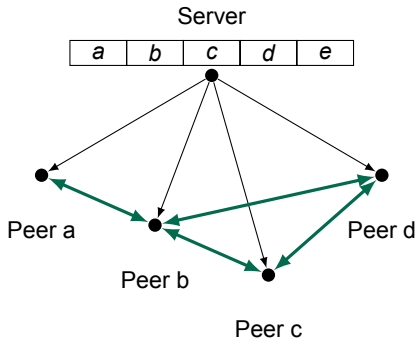
Gkantsidis and Rodriguez,
IEEE INFOCOM, 2005



Avalanche: Large-scale Content Distribution

Gkantsidis and Rodriguez,
IEEE INFOCOM, 2005

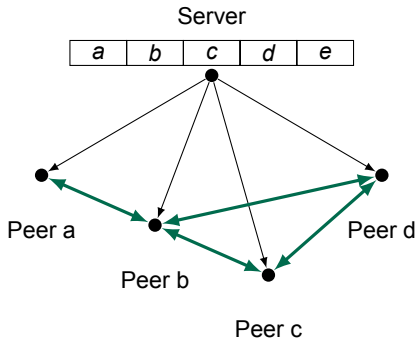
- ▶ Peer-assisted file delivery



Avalanche: Large-scale Content Distribution

Gkantsidis and Rodriguez,
IEEE INFOCOM, 2005

- ▶ Peer-assisted file delivery
- ▶ Challenges addressed by using network coding
 - ▶ Scheduling
 - ▶ Obtaining the last piece
- ▶ Used in practice:
Microsoft Secure Content
Downloader



Distributed Storage

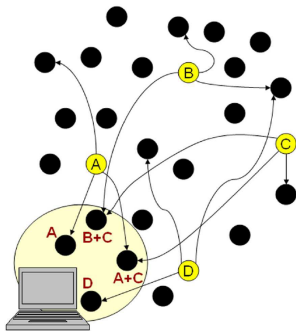
Dimakis et al., IEEE IT, vol. 56,
no. 9, 2010

Challenges:

- ▶ Node failures
- ▶ Recover data from any subset of nodes
- ▶ Repair bandwidth

Applications:

- ▶ Sensor networks
- ▶ Clouds



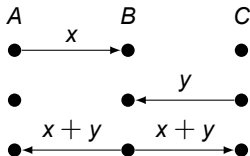
Dimakis et al., IEEE/ACM TON, vol. 14,
no. SI, 2006

Security against Eavesdropping

- ▶ Attacker overhearing transmissions by relay B
- ▶ Bits x and y uniform:

$$P(x = 1) = P(x = 0) = 1/2,$$

$$P(y = 1) = P(y = 0) = 1/2.$$



The bit $z = x + y$ does not give any information on x or y .

Potential Applications

- ▶ Increase efficiency, leading to
 - ▶ higher throughput
 - ▶ improved robustness
 - ▶ less infrastructure
 - ▶ lower energy consumption
 - ▶ lower delay

Potential Applications

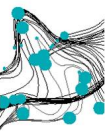
- ▶ Increase efficiency, leading to
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- ▶ Do network tomography
- ▶ Improve P2P applications

Potential Applications

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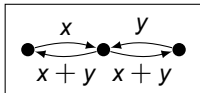
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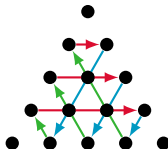
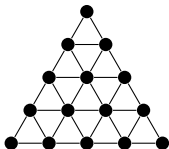
Energy Savings in Wireless Networks

Line Network



Large network: Energy savings approach factor 2.

Hexagonal Lattice



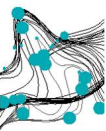
Theorem

Energy savings of factor 3 can be achieved.

- ▶ Extensions: Factor 4 using another configuration, factor 6 in 3D.
- ▶ Joint work with: Jos Weber, Ryutaroh Matsumoto and Tomohiko Uyematsu.

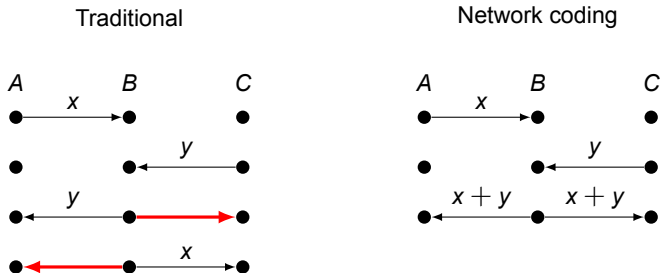
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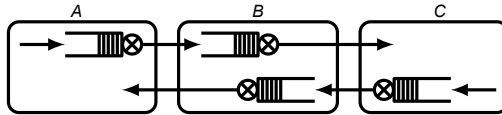
Stochastic Arrival and Transmit Times

Wireless Information Exchange using a Single Relay



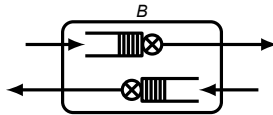
- ▶ Make **more efficient** use of resources.
- ▶ Reduce energy consumption and delay by decreasing number of transmissions.

Stochastic Arrival and Service Times



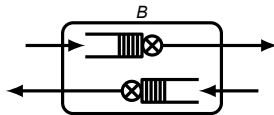
- Keep a queue for each connection

Stochastic Arrival and Service Times



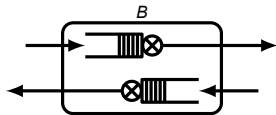
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Stochastic Arrival and Service Times



- ▶ Keep a queue for each connection
- ▶ Sending a coded packet ($x + y$) corresponds to removing a packet from both queues
- ▶ Queues will sometimes be empty
- ▶ What to do if we have only one packet?

Stochastic Arrival and Service Times



- ▶ Keep a queue for each connection
- ▶ Sending a coded packet ($x + y$) corresponds to removing a packet from both queues
- ▶ Queues will sometimes be empty
- ▶ What to do if we have only one packet?

Wait for another packet

Delay \uparrow

Energy \downarrow

Tradeoff

Transmit an uncoded packet

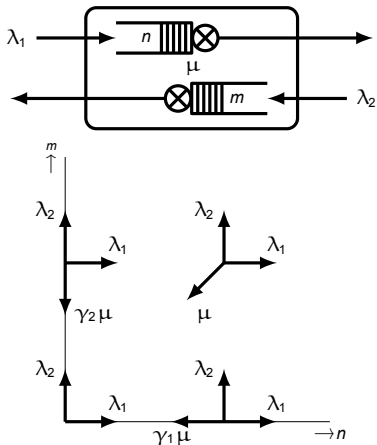
Delay \downarrow

Energy \uparrow

Model

- ▶ Continuous-time Markov chain
- ▶ Two queues, N packets in queue 1, M packets in queue 2.
- ▶ Poisson arrivals at rate λ_i
- ▶ Exponential service time, rate μ
- ▶ γ_i : Probability of transmitting uncoded packet from queue i

Joint work with Richard Boucherie and Jan-Kees van Ommeren.



Stability

Theorem

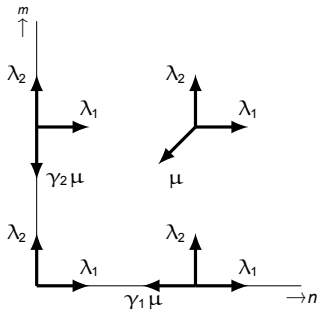
The process is ergodic if and only if

$$\lambda_1 < \mu, \quad \lambda_2 < \mu,$$

$$\gamma_1 > \frac{\lambda_1 - \lambda_2}{\mu - \lambda_2}, \quad \gamma_2 > \frac{\lambda_2 - \lambda_1}{\mu - \lambda_1}.$$

Example

- ▶ $\lambda_1 = \lambda_2: \gamma_1 > 0, \gamma_2 > 0.$

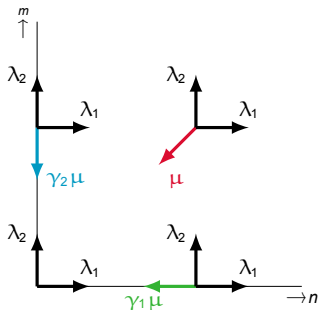


Performance Measures

- ▶ Delay, D_1 and D_2
 - ▶ Little's law
 - ▶ $D_1 = \frac{1}{\lambda_1} \mathbb{E}[N]$, $D_2 = \frac{1}{\lambda_2} \mathbb{E}[M]$

Performance Measures

- ▶ Delay, D_1 and D_2
 - ▶ Little's law
 - ▶ $D_1 = \frac{1}{\lambda_1} \mathbb{E}[N]$, $D_2 = \frac{1}{\lambda_2} \mathbb{E}[M]$
- ▶ Energy consumption, C
 - ▶ Cost μ per unit time if transmitting
 - ▶ if one queue is empty, transmit with probability γ_1, γ_2
 - ▶ $C = \mathbb{E}[e(N, M)]$
 - ▶ $e(n, m) = \gamma_1 \mu \mathbf{1}_{\{n>0, m=0\}} + \gamma_2 \mu \mathbf{1}_{\{n=0, m>0\}} + \mu \mathbf{1}_{\{n>0, m>0\}}$



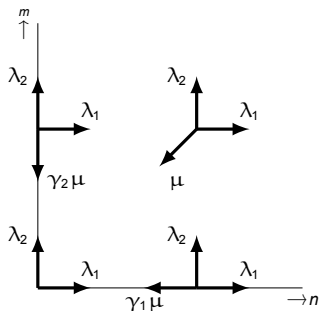
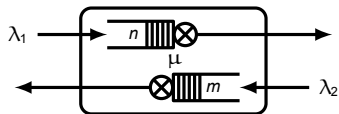
Queueing Networks with Positive and Negative Customers

- ▶ Poisson arrivals
- ▶ Queues with (independent) exponential service
- ▶ Customers, after receiving service,
 - ▶ leave network, or
 - ▶ enter another queue, or
 - ▶ **remove a customer from another queue and together leave the network**

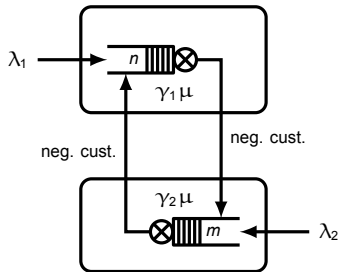
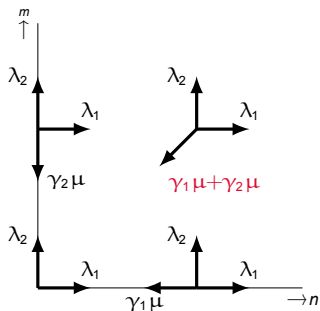
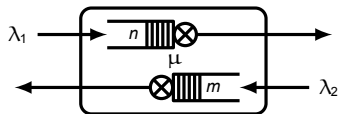
Theorem (Gelenbe 1991)

Queueing networks with positive and negative customers have geometric product-form stationary distributions.

Information Exchange if $\gamma_1 + \gamma_2 = 1$



Information Exchange if $\gamma_1 + \gamma_2 = 1$



Stationary Distribution for $\gamma_1 + \gamma_2 = 1$

Theorem

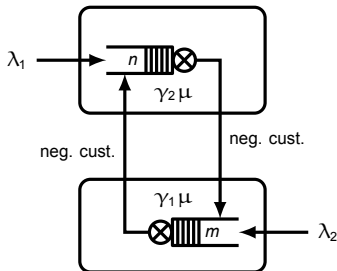
Let $\gamma_1 + \gamma_2 = 1$. The stationary distribution $\pi(n, m)$ is given by

$$\pi(n, m) = (1 - \sigma_1)\sigma_1^n(1 - \sigma_2)\sigma_2^m,$$

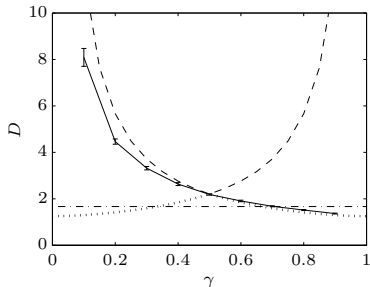
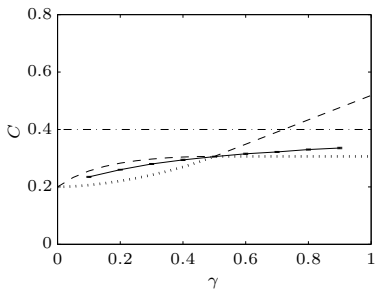
where σ_1, σ_2 are the solution in $(0, 1)$ of

$$\sigma_1 = \frac{\lambda_1}{\gamma_1\mu + \gamma_2\mu\sigma_2},$$

$$\sigma_2 = \frac{\lambda_2}{\gamma_2\mu + \gamma_1\mu\sigma_1}.$$



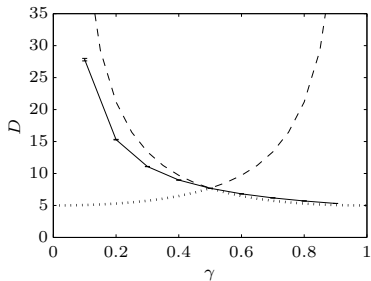
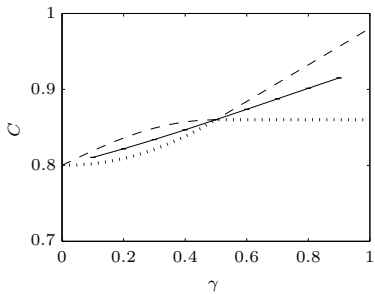
Numerical Example: Symmetrical, low load



- ▶ $\mu = 1, \lambda_1 = \lambda_2 = 0.2, \gamma_1 = \gamma_2 = \gamma$
- ▶ C : energy consumption, D : delay of first connection
- ▶ Solid lines: simulation results, Dashed–dotted lines: uncoded system

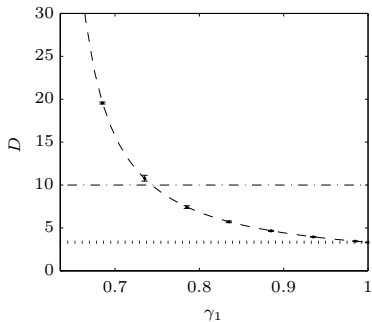
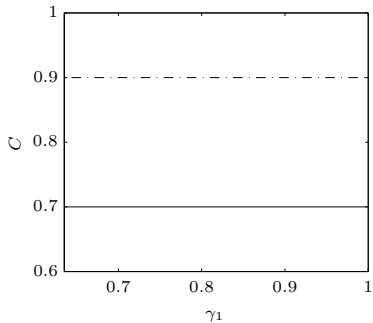
Network coding does not improve delay for low values of γ .

Numerical Example: Symmetrical, high load



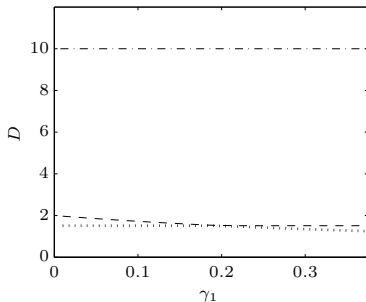
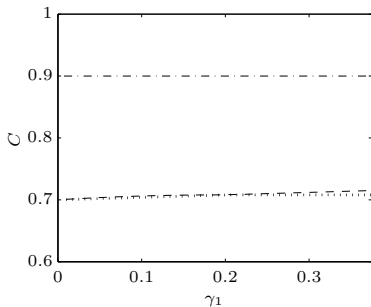
► $\mu = 1, \lambda_1 = \lambda_2 = 0.8, \gamma_1 = \gamma_2 = \gamma$

Numerical Example: $\lambda_1 > \lambda_2$

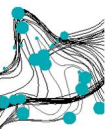


► $\mu = 1, \lambda_1 = 0.7, \lambda_2 = 0.2, \gamma_2 = 0.8$

Numerical Example: $\lambda_1 < \lambda_2$



► $\mu = 1, \lambda_1 = 0.2, \lambda_2 = 0.7, \gamma_2 = 0$



Outlook:

Extend current performance analysis to

- ▶ Better approximations
- ▶ Other performance measures
- ▶ Larger networks

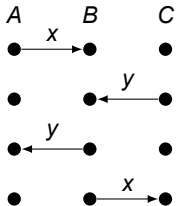
And...

- ▶ Develop protocols
- ▶ Implement!

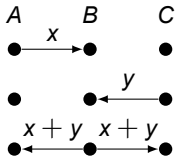


Exploiting Superposition

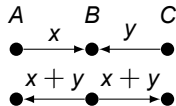
Traditional



Network coding



Computation Coding



- ▶ Computation coding / Physical layer network coding / Analog network coding.
- ▶ Perform channel coding to reliably decode functions of messages.